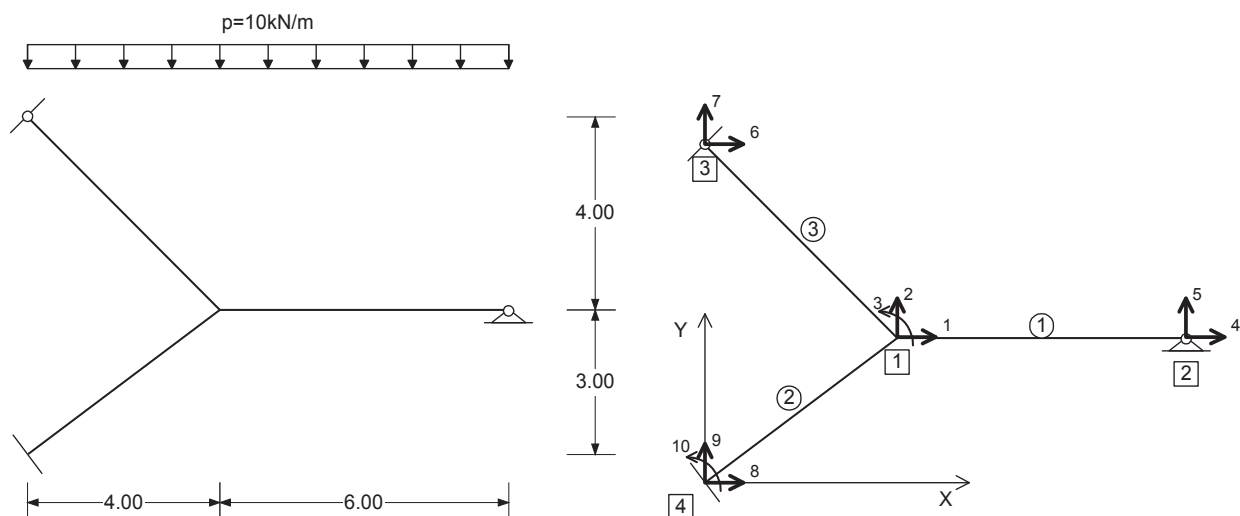


TAČNA METODA DEFORMACIJE



$$EI = \text{const.}; F/l = 50 \cdot 1/\text{m}^2$$

$$n = 2k + m + z_u = 8 + 1 + 1 = 10$$

$$o = z_o + z_u = 6 + 1 = 7$$

$$s = 10 - 7 = 3 \quad \text{nepoznata pomeranja } 1, 2, 3$$

štap	i	k	$\cos \alpha$	$\sin \alpha$	F/l	l
1	1	2	1	0	50	6.0
2	4	1	0.8	0.6	50	5.0
3	1	3	-0.707	0.707	50	5.657

MATRICE TRANSFORMACIJE

$$\text{štap 1} \quad T_1 = I$$

$$\text{štap 2} \quad T_2 = \begin{bmatrix} 0.8 & 0.6 & & \\ -0.6 & 0.8 & & \\ & & 1 & \\ & & & 0.8 & 0.6 \\ & & & -0.6 & 0.8 \\ & & & & & 1 \end{bmatrix}$$

$$\text{štap 3} \quad T_3 = \begin{bmatrix} -0.707 & 0.707 & & \\ -0.707 & -0.707 & & \\ & & 1 & \\ & & & -0.707 & 0.707 \\ & & & -0.707 & -0.707 \end{bmatrix}$$

MATRICE KRUTOSTI ŠTAPOVA U LOKALNOM KOORDINATNOM SISITEMU

$$\text{štap 1} \quad k_1 = EI \cdot \begin{bmatrix} F/l \cdot I & & & -F/l \cdot I \\ & 3/l^3 & 3/l^2 & -3/l^3 \\ & & 3/l & -3/l^2 \\ & & & F/l \cdot I \\ & & & & 3/l^3 \end{bmatrix} = EI \cdot \begin{bmatrix} 8.333 & & & -8.333 \\ & 0.0139 & 0.0833 & -0.0139 \\ & & 0.5 & -0.0833 \\ & & & 8.333 \\ & & & & 0.0139 \end{bmatrix}$$

$$\text{štap 2} \quad k_2 = EI \cdot \begin{bmatrix} F/l \cdot I & & & -F/l \cdot I \\ & 12/l^3 & 6/l^2 & -12/l^3 & 6/l^2 \\ & & 4/l & -6/l^2 & 2/l \\ & & & F/l \cdot I & 12/l^3 & -6/l^2 \\ & & & & 12/l^3 & -6/l^2 \\ & & & & & 4/l \end{bmatrix} = EI \cdot \begin{bmatrix} 10 & & & -10 \\ & 0.096 & 0.24 & -0.096 & 0.24 \\ & & 0.8 & -0.24 & 0.4 \\ & & & 10 & 0.096 & -0.24 \\ & & & & 10 & 0.096 & -0.24 \\ & & & & & 0.8 \end{bmatrix}$$

$$\text{štap 3} \quad k_3 = EI \cdot \begin{bmatrix} 8.8388 & & & -8.8388 \\ & 0.0166 & 0.0938 & -0.0166 \\ & & 0.5303 & -0.0938 \\ & & & 8.8388 \\ & & & & 0.0166 \end{bmatrix}$$

MATRICE KRUTOSTI ŠTAPOVA U GLOBALNOM KOORDINATNOM SISTEMU $k_j^* = T_j^t \cdot k_j \cdot T_j$

$$\text{štap 1} \quad k_1^* = k_1 = EI \cdot \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 8.333 & & & -8.333 & \\ & 0.0139 & 0.0833 & & -0.0139 \\ & & 0.5 & & -0.0833 \\ & & & 8.333 & \\ & & & & 0.0139 \end{bmatrix} \end{matrix}$$

$$\text{štap 2} \quad k_2^* = T_2^t \cdot k_2 \cdot T_2 = EI \cdot \begin{matrix} & \begin{matrix} 8 & 9 & 10 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 8 \\ 9 \\ 10 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 6.4346 & 4.7539 & -0.144 & -6.4346 & -4.7539 & -0.144 \\ & 3.6614 & 0.192 & -4.7539 & -3.6614 & 0.192 \\ & & 0.8 & 0.144 & -0.192 & 0.4 \\ & & & 6.4346 & 4.7539 & 0.144 \\ & & & & 3.6614 & -0.192 \\ & & & & & 0.8 \end{bmatrix} \end{matrix}$$

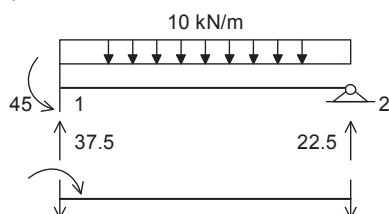
$$\text{štap 3} \quad k_3^* = T_3^t \cdot k_3 \cdot T_3 = EI \cdot \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} 4.4277 & -4.4111 & -0.0663 & -4.4277 & 4.4111 \\ & 4.4277 & -0.0663 & 4.4111 & -4.4277 \\ & & 0.5303 & 0.0663 & 0.0663 \\ & & & 4.4277 & -4.4111 \\ & & & & 4.4277 \end{bmatrix} \end{matrix}$$

MATRICA KRUTOSTI SISTEMA

$$K = EI \cdot \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} & \begin{bmatrix} 19.1956 & 0.3428 & 0.0777 & -8.333 & & -4.4277 & 4.4111 & -6.4346 & -4.7539 & 0.144 \\ & 8.1030 & -0.175 & & 0.0139 & 4.4111 & -4.4277 & -4.7539 & -3.6614 & -0.192 \\ & & 1.8303 & & -0.0833 & 0.0663 & 0.0663 & -0.144 & 0.192 & 0.4 \\ & & & 8.333 & & & & & & \\ & & & & 0.0139 & & & & & \\ & & & & & 4.4277 & -4.4111 & & & \\ & & & & & & 4.4277 & & & \\ & & & & & & & 6.4346 & 4.7539 & -0.144 \\ & & & & & & & & 3.6614 & 0.192 \\ & & & & & & & & & 0.8 \end{bmatrix} \end{matrix}$$

VEKTOR EKVALENTNOG ČVORNOG OPTEREĆENJA

štap 1



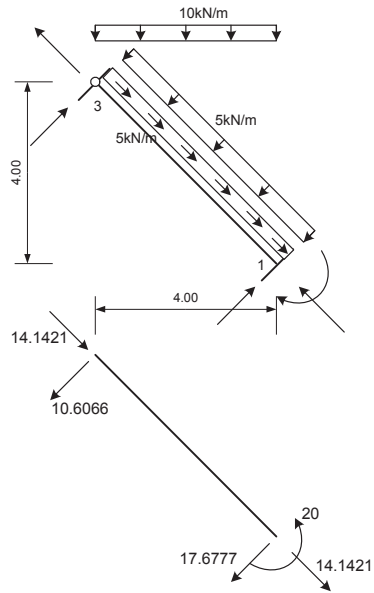
$$M_{12} = \frac{10 \cdot 6^2}{8} = 45 \text{ kNm}$$

$$T_{12} = (10 \cdot 6 \cdot 3 + 45) \cdot \frac{1}{6} = 37.5 \text{ kN}$$

$$T_{21} = (10 \cdot 6 \cdot 3 - 45) \cdot \frac{1}{6} = 22.5 \text{ kN}$$

$$Q_1 = Q_1^* = \begin{bmatrix} 0 \\ -37.5 \\ -45 \\ 0 \\ -22.5 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}$$

štap 3



$$M_{13} = \frac{10 \cdot 4^2}{8}$$

$$T_{13} = (20 + 10 \cdot 4 \cdot 2) \cdot \frac{1}{4 \cdot \sqrt{2}} = 17.6777$$

$$T_{31} = (-20 + 10 \cdot 4 \cdot 2) \cdot \frac{1}{4 \cdot \sqrt{2}} = 10.6066$$

$$N_{13} = N_{31} = 5 \cdot 4 \cdot \sqrt{2} \cdot \frac{1}{2} = 14.1421$$

$$Q_3 = \begin{bmatrix} -14.1421 \\ 17.6777 \\ 20 \\ -14.1421 \\ 10.6066 \end{bmatrix} \quad Q_3^* = \begin{bmatrix} -2.5 \\ -22.5 \\ 20 \\ 2.5 \\ -17.5 \end{bmatrix}$$

VEKTOR NEPOZNATOG EKIVALENTNOG ČVORNOG OPTEREĆENJA

$$Q_{1,s}^* = \begin{bmatrix} 0 \\ -37.5 \\ -45 \end{bmatrix} \quad Q_{3,s}^* = \begin{bmatrix} -2.5 \\ -22.5 \\ 20 \end{bmatrix} \Rightarrow Q_s^* = \begin{bmatrix} -2.5 \\ -60 \\ -25 \end{bmatrix}$$

VEKTOR NEPOZNATIH POMERANJA

$$\left. \begin{aligned} q_s^* &= K_{ss}^{-1} \cdot S_s^* + K_{ss}^{-1} \cdot K_{so} \cdot q_o^* \\ q_o^* &= 0 \end{aligned} \right\} \Rightarrow q_s^* = K_{ss}^{-1} \cdot S_s^* \quad \left. \begin{aligned} S_s^* &= P_s^* + Q_s^* \\ P_s^* &= 0 \end{aligned} \right\} \Rightarrow S_s^* = Q_s^*$$

$$q_s^* = \frac{1}{EI} \begin{bmatrix} 0.052145 & -0.00226 & -0.00243 \\ -0.00226 & 0.123764 & 0.011929 \\ -0.00243 & 0.011929 & 0.547602 \end{bmatrix} \cdot \begin{bmatrix} -2.5 \\ -60 \\ -25 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.065887 \\ -7.71844 \\ -14.3997 \end{bmatrix}$$

SILE NA KRAJEVIMA ŠTAPOVA

$$R_j^* = k_j^* \cdot q_j^* - Q_j^*; \quad R_j = k_j \cdot q_j - Q_j; \quad q_j = T \cdot q_j^*$$

$$\text{štap 1} \quad R_1^* = k_1^* \cdot q_1^* - Q_1^* = k_1^* \cdot \begin{bmatrix} 0.065887 \\ -7.71844 \\ -14.3997 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -37.5 \\ -45 \\ 0 \\ -22.5 \end{bmatrix} = \begin{bmatrix} 0.549 \\ 36.193 \\ 37.157 \\ -0.549 \\ 23.807 \end{bmatrix}$$

$$\text{štap 2} \quad R_2 = k_2 \cdot q_2 - Q_2 = k_2 \cdot T_2 \cdot q_2^* = k_2 \cdot T_2 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.065887 \\ -7.71844 \\ -14.3997 \end{bmatrix} = \begin{bmatrix} 45.783 \\ -2.859 \\ -4.268 \\ -45.783 \\ 2.859 \\ -10.028 \end{bmatrix}$$

štap 3

$$R_3 = k_3 \cdot q_3 - Q_3 = k_3 \cdot T_3 \cdot q_3^* - Q_3 = k_3 \cdot T_3 \cdot \begin{bmatrix} 0.065887 \\ -7.71844 \\ -14.3997 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -14.1421 \\ 17.6777 \\ 20 \\ -14.1421 \\ 10.6066 \end{bmatrix} = \begin{bmatrix} -32.0258 \\ -18.9383 \\ -27.1287 \\ 60.31002 \\ -9.3461 \end{bmatrix}$$

